Methods to Determine Torsion Stiffness in an Automotive Chassis

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ABSTRACT

This paper reviews three different approaches to determine the torsion stiffness of an automotive chassis and presents a Finite Element Analysis based method to estimate a vehicle's torsion stiffness. As a case study, a typical sedan model is analyzed and the implementation of the presented methodology is demonstrated. Also presented is a justification of assuming linear torsion stiffness when analyzing the angle of twist. The presented methodology can be utilized in various aspects of vehicle structural design.

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1 INTRODUCTION

Torsion stiffness is an important characteristic in chassis design with an impact on the ride and comfort as well as the performance of the vehicle [5],[6],[10]. With this in mind the goal of design is to increase the torsion stiffness without significantly increasing the weight of the chassis. In order to achieve this goal this paper presents various methods to determine the torsion stiffness at different stages in the development process. The different methods include an analytical method, simulation method and experimental method. Also the reasons for seeking a linear torsion stiffness value are presented. A sample simulation is performed using Finite Element Analysis (FEA).

2 ANALYTICAL METHOD

Determining torsion stiffness based only on the geometry of the chassis can be difficult for a vehicle given the complex geometries that are commonly found [3]. However by expanding on the principles of solid mechanics and making some simplifications a method can be developed to give an approximate value for the chassis. If the applied torque (T) is related to the angle of twist of a chassis (φ) through the following equation:

\[ T = \frac{JG\phi}{L} = K_T\phi \]  

Eqn. 1
Where:
J= Polar moment of inertia
G= Material shear modulus of elasticity
L= Characteristic length of cross section

Since a chassis has multiple, complex cross sections it is necessary to consider the chassis as a series of different cross-sections fixed together. With this assumption and the superposition method it is possible to analytically determine the overall torsion stiffness for the vehicle’s structure by superimposing the individual stiffness values of the components [4]. The primary difficulty in analytically determining the torsion stiffness is due to the complex geometries found in a vehicle chassis and the unknown nature of how these geometries will deform when twisted [7]. For these cross sections it can be assumed that the cross sections remain planar or that they remain undistorted in their own plane. This leads to some inaccuracies in the value obtained however the analysis should be a good initial approximation.

3 SIMULATION METHOD
The simulation method is based on previously conducted studies where FEA is performed on the chassis [9]. A case study is presented later to validate the method described here. In this method equal and opposite loads are applied at the front suspension mounting locations while the rear mounting locations remain fixed. The equations used to determine the torsion stiffness is based on the total deflection of the mounting locations. The torsion stiffness is calculated using the following equations.

\[ K_T = \frac{T}{\varphi} = \frac{FB}{(\varphi_d + \varphi_p)} \]  
Eqn. 2

Where:

\[ \varphi_d = \tan^{-1}\left(\frac{v_d}{B/2}\right) \]  
Eqn. 3

\[ \varphi_p = \tan^{-1}\left(\frac{v_p}{B/2}\right) \]  
Eqn. 4

In the above equations the torque, T, is represented by the vertical force applied at the mounting locations, F, and the track width of the vehicle, B. The angular deflections (\( \theta_d \) and \( \theta_p \)) are based on the vertical deflections for the driver (\( v_d \)) and passenger (\( v_p \)) sides of the vehicle, as well as the track width. The angular deflections should be similar but are not necessarily exactly equal due to small differences in the geometry of the vehicle as well as small differences in where the loads are applied on the vehicle mesh.

4 EXPERIMENTAL METHOD
The experimental method to determine torsion stiffness is similar to the simulation method described above [8]. In this method linear jack-screw actuators are used to apply vertical deflections at the front suspension mounting locations in gradual increments. The rear mounting locations are held fixed and load cells are placed at each location under each jack stands in order to determine the force applied. Dial indicators are used to measure the deflections at various points along the chassis. The torsion stiffness is found from the following equations:

\[ T = \left(\frac{|R_i| + |R_i|}{2}\right) L_s \]  
Eqn. 5

\[ \theta_r = \frac{|\delta_i| + |\delta_i|}{L_r} \]  
Eqn. 6

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In the above equation the torque, $T$, is based on the reaction forces ($R_r$ and $R_l$) for the right and left sides respectively, as well as the lateral distance between scales. The angular deflection, $\theta$, is again based on the vertical deflections at the right and left wheels respectively ($\delta_r$ and $\delta_l$) which are found from the measurements on the jack screw actuators. The torsion stiffness, $K$, is based on the torque and angular deflection and is found at several increments of the jack screw actuators. An average value representing the actual torsion stiffness is then found using a least squares regression.

Other studies have been conducted using this method and it is commonly used for testing a race chassis. A previous test performed using a structure of calculated torsion stiffness found that the experimental method described here yielded a result that was 6% larger than the known, calculated value with 95% confidence [8].

5 LINEARITY OF TORSION STIFFNESS

It is assumed that the torsion stiffness described above follows a linear curve where the stiffness can be accurately determined as the slope of a function involving the torque and the angle of twist. This assumption is based on the fact that the angle of twist is related to the torque through geometric and material properties only as shown in Equation 1.

As can be seen in this equation, if the material and geometric properties are constant then the torsion stiffness will be constant as well. This is analogous to that of a linear spring with a linear spring rate. It is possible that the torsion stiffness could end up nonlinear, in which case a better model would be required to accurately predict the torsion characteristics. In order to determine the nature of the torsion stiffness the test must be performed for a variety of applied torques and the deflections measured. The resulting data can be analyzed to study the linearity of the torsion stiffness function. The slope of the angular deflection vs. torque plot will represent the torsion stiffness if it is linear. It can be seen that linear torsion stiffness is preferred in order to more accurately predict the chassis behavior.

6 CASE STUDY

The simulation method described above was applied to a simplified sedan body that was structurally analyzed using the Simple Structural Surfaces (SSS) method [2]. The SSS method is a simple analytical approach for initial analyses of a preliminary design concept. Structural feasibility of the presented sedan design was confirmed using SSS analysis. As part of the case study the results from this analysis were included here. The model used is the same as that utilized for the simulation. The following is a brief background of the method used.

The Simple Structural Surface method is used to analyze simple structures using thin plates as structural members. There are two key assumptions made when analyzing a structure. The first is that the structure is statically determinant [2]. This assumption limits the accuracy, especially in vehicle design where a number of redundant structures are used. The second assumption is that a sheet is unable to react out of plane loads, that is to say they have zero stiffness to loads applied perpendicular to the surface. When analyzing a vehicle a systematic approach is used where sheets are analyzed one at a time starting with sheets containing the input loads, which were calculated separately. It should be noted that the front clip, where the engine and front suspension are, has been omitted because there are not edge loads there. It acts as an input for the vehicle structure. The end result will be the edge loads of each sheet, as labeled in the figure below.
These loads can then be compared with the deflections seen in the simulation results and a relation can be found. The load results are shown in the table below.

<table>
<thead>
<tr>
<th>Edge</th>
<th>Load (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q_1</td>
<td>2835</td>
</tr>
<tr>
<td>Q_2</td>
<td>2358</td>
</tr>
<tr>
<td>Q_3</td>
<td>189</td>
</tr>
<tr>
<td>Q_4</td>
<td>1538</td>
</tr>
<tr>
<td>Q_5</td>
<td>2363</td>
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<td>1343</td>
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<td>Q_7</td>
<td>1040</td>
</tr>
<tr>
<td>Q_8</td>
<td>-280</td>
</tr>
<tr>
<td>Q_9</td>
<td>3024</td>
</tr>
<tr>
<td>Q_10</td>
<td>405</td>
</tr>
<tr>
<td>Q_11</td>
<td>2741</td>
</tr>
</tbody>
</table>

Table 1: SSS Torsion Results.

As can be seen the edge loads vary greatly. As a comparison between these results and the results obtained through model simulation it is worth noting that the sheets with primarily vertical loads such as the windscreen have generally smaller loads. This is likely due to the fact that those loads are being transferred to the main components, which are the roof and floor. A vertical load in these sheets would be considered out of plane and therefore the edge loads on the longitudinal side have to be substantially higher to properly react the forces. The one area this does not hold true, when comparing SSS with simulation, is the trunk floor. However since in the simulation the rear suspension is held fixed the decreased deflections would be due to their proximity to the fixed location.

Following the SSS analysis the model was used as a basis for validation of the simulation method described above. The vehicle model that was used for the case study is shown below. As can be seen there are a total of 18 panels in the half car model shown.
The vehicle shown has an overall length of 4.4m, a width of 1.5m and a maximum height of 1.35m (Fig. 2). The loading conditions that are applied to the car model are shown below.
The applied loads are applied at the approximate location of the suspension mounting points. The loads that are applied are equal in magnitude but opposite in direction. Using this loading condition will create the case of pure torsion. The fixed constraint that is used is applied in the rear wheel well, again approximately where the rear suspension would be mounted. The finite element model uses a quadrilateral shell element type with Aluminum 6061 as the material. The surfaces all have the same 2mm thickness. The solution uses FEA software as the solver with a single constraint structural analysis. The mesh is a 2D mesh with four nodes for each element. The size of each element was generated using the Automatic Element Size feature with less than 100 mm dimensions giving a total of 2603 elements.

In order to conduct the simulation process, equal and opposite loads are applied at the approximate location of the suspension mounting points. The approximate location of rear suspension mounts is held fixed for all degrees of freedom.

A typical result of performing the simulation is shown below.

As can be seen the front of the vehicle has been twisted relative to the rear. Using the measurement tools provided in the FEA package the vertical deflections can be approximated at the location the loads are applied.

FEA of the model was performed for the applied torques ranging from 1500Nm to 3750Nm with increments of 375Nm. The loads applied were increased incrementally and a graph of applied torque compared to the angle of rotation was generated. The slope of this graph is the overall torsion stiffness of the body. Equations (2), (3) and (4) were used to determine the torsion stiffness of the body.
After performing a total of seven trials the overall torsion stiffness was found to be 3000Nm/rad. The result obtained is a reasonable value given that the model used is simplified compared to an actual vehicle and has numerous structural members missing such as cross members and all panels are of uniform thickness and material. The stiffness target for typical vehicles is between 10-18kNm/rad [1].

7 RESULTS

The complete table of results is shown in the appendix. After performing multiple simulations a plot was produced with a fitted line (Figure 5). The fitted line was used to determine the torsion stiffness. The residuals values were also plotted for different angles of twist. The values shown are the absolute values of the fitting errors (Figure 6). The absolute error varies between only ±80Nm so the percent change of the fitting error, which is an indicator of the validity of the linearity assumption, will be:

\[
\frac{160}{3750} \times 100\% = 4.3\%
\]

As can be seen the torsion stiffness is linear. The simulation results may appear bilinear however that is due to having one result slightly further away from a best fit line and the plotting software connecting the data points with a straight line. The simulation results are actually discrete values that are then connected by a thick line for clarity.
The above value presents the accuracy of the linear interpolation and can therefore be used to predict the overall torsion stiffness of the case study. The errors present can be partially attributed to errors in the parameters of finite element analyses including mesh configuration, element size and element type. If the mesh were refined a reduction in error should occur with a corresponding increase in computation time. It would be possible to determine the amount of error that results from the mesh size by performing multiple runs with varying mesh sizes and calculating the different values for fitting error.

8 CONCLUSION

This paper reviews three different approaches; analytical, simulation and experimental, to determine the torsion stiffness of an automotive chassis at the different stages of development. A comparison of two Finite Element based methods is also conducted. The goal of the analytical method would be to determine the stiffness based only on the geometry but could prove too costly in terms of time as a large number of long calculations could be required. The presented FEA-based method is a more practical method that can be employed in industry. This method uses finite element analysis to apply loads and measure the resulting deflections. Overall any of the methods presented could be applied in order to determine the torsion stiffness but it is suggested that a combination of methods, especially simulation and experimental methods, in order to verify the results. While increasing the torsion stiffness is an important goal in automotive design a more practical measurement would be a function involving the torsion stiffness and vehicle mass to ensure the increases in torsion stiffness will not greatly increase the weight of the vehicle.

REFERENCES